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RECOVERY OF THE TEMPERATURE OF THE SURFACE OF A COPPER ELECTRODE IN ELECTRICAL CONTACT SINTERING

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On the basis of solution of the inverse problem of heat conduction by a method of quasi-inversion we developed an algorithm for recovery of the temperature of the surface of contact of a copper electrode with sintered metal powder by results of temperature measurements at a certain depth in the electrode. Results of the numerical experiment which indicate the efficiency of the algorithm are presented.

The process of electrical contact sintering of metal powders is used in repair production for renovation and manufacture of parts of agricultural machines. For optimization of the regimes of the process of electrical contact sintering of metal powders we require data on the nonstationary temperatures of surfaces of electrodes [1, 2]. At the same time, it is known that measurement of the body surface temperature is accompanied by a considerable methodological error. Therefore, thermometry data obtained at a certain depth are used instead of direct measurement of temperature on the studied surface. Then these data are recalculated to the surface temperature by solving an inverse problem of heat conduction. This is an ill-posed problem; therefore, in the present work, in order to solve the problem, we used a regularizing algorithm based on the method of quasi-inversion [3, 4]. The inverse problem is formulated in the form of the Cauchy problem. Results of the numerical solution of a model problem of monotonic heating of a copper electrode are used as data of temperature measurements at different depths (Fig. 1).

The problem of recovery of the temperature of the copper electrode is formulated as follows. It is necessary to find an expanded vector of temperatures and temperature gradients $\{q, T\}$ by solving the Cauchy problem written as

$$\frac{\partial T}{\partial t} = -a\frac{\partial q}{\partial x}, \quad 0 < x < x_1, \quad 0 < t < t_{\mathrm{f}};$$

$$\frac{\partial T}{\partial x} = -q, \quad 0 < x < x_1, \quad 0 < t < t_{\mathrm{f}}; \tag{1}$$

$$T(0, t) = T_0(t), \quad \frac{\partial T}{\partial x} = -q_0(t), \quad x = 0, \quad 0 < t < t_{\mathrm{f}}.$$

To solve (1) we use a finite-difference method. In the initial region of variation of continuous arguments $G = \{(x, t) : 0 < t < t_{\mathrm{f}}, 0 < x < x_1\}$ we construct a rectangular grid $G^{h,\tau} = \{t_i = t_{i-1} + \tau, i = 1, 2, ..., N; x_j = x_{j-1} + h, j = 1, 2, ..., M\}$.

Replacing the derivatives by finite-difference relations, we obtain the following system of computational equations:

$$q_{i+1}^{1} = -\frac{h}{a\tau}(T_i^2 - T_i^1) - q_i^1, \quad j = 1;$$

$$q_{i+1}^{j} = -\frac{h}{2a\tau}(T_i^{j+1} - T_i^{j-1}) - q_i^j, \quad j = 2, ..., M-1;$$

$$q_{i+1}^{M} = -\frac{h}{a\tau}(T_i^{M} - T_i^{M-1}) - q_i^M, \quad j = M;$$

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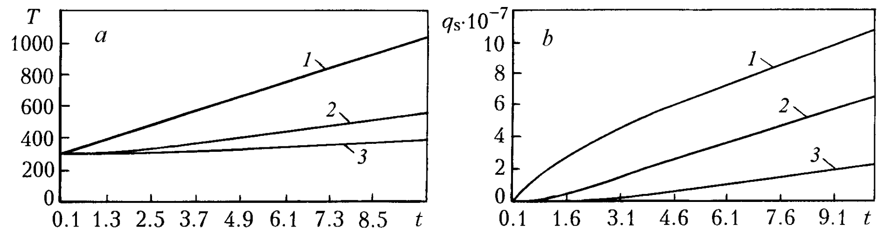

Fig. 1. Dependences of temperature (a) and heat-flux density (b) on time: 1) on the surface of the electrode; 2) at a depth of 0.1 mm; 3) 0.2. T, K; q_s, W/m^2; t, sec.

$$T_{i+1}^{j} = T_i^j - hq_i^j\,, \quad j = 1, 2, ..., M\,. \tag{2}$$

Using the maximum principle, we can show instability of the computational algorithm (2). To construct a stable algorithm, we use the method of quasi-inversion:

$$\begin{gathered}
\frac{\partial q_\alpha}{\partial t} = -\frac{1}{a}\frac{\partial T_\alpha}{\partial x} - \alpha q_\alpha\,, \quad 0 < t < t_{\hat{e}}\,, \quad 0 < x < x_1\,; \\
\frac{\partial T_\alpha}{\partial t} = \frac{\alpha}{a^2}\frac{\partial^2 T_\alpha}{\partial x^2} - q_\alpha\,, \quad 0 < t < t_{\hat{e}}\,, \quad 0 < x < x_1\,; \\
q_\alpha = q_0\,, \quad T_\alpha = T_0\,, \quad t = 0\,, \quad 0 < x < x_1\,.
\end{gathered} \tag{3}$$

After substitution of partial derivatives by finite-difference relations we obtain the following computational relations (the subscript α is omitted for the sake of simplicity):

$$\begin{gathered}
q_{i+1}^1 = -\frac{\tau}{ah}\,(T_i^2 - T_i^1) - (\alpha - 1)\,q_i^1\,, \quad j = 1\,; \\
q_{i+1}^j = -\frac{\tau}{2ah}\,(T_i^{j+1} - T_i^{j-1}) - (\alpha - 1)\,q_i^j\,, \quad j = 2, ..., M-1\,; \\
q_{i+1}^M = -\frac{\tau}{ah}\,(T_i^M - T_i^{M-1}) - (\alpha - 1)\,q_i^M\,, \quad j = M\,; \\
T_{i+1}^j = \frac{\alpha\tau}{a^2h^2}\,(T_i^{j+1} - 2T_i^j + T_i^{j-1}) + T_i^j - \tau q_i^j\,, \quad j = 2, ..., M-1\,; \\
T_{i+1}^1 = \frac{\alpha\tau}{a^2h^2}\,(T_i^3 - 2T_i^2 + T_i^1) + T_i^1 - \tau q_i^1\,, \quad j = 1\,; \\
T_{i+1}^M = \frac{\alpha\tau}{a^2h^2}\,(T_i^M - 2T_i^{M-1} + T_i^{M-2}) + T_i^M - \tau q_i^M\,, \quad j = M\,.
\end{gathered} \tag{4}$$

Peforming calculations by formulas (4) successively for each i, we find a regularized solution which depends on the parameter α ($\alpha > 0$).

In order to refine the numerical algorithm of solution of the problem posed, we conducted a numerical experiment. The temperature and its gradient at a depth of 0.1 mm were taken as initial data for the calculations. Calculations inside the sample were performed by formulas (2) (without regularization) and by (4) using the method of

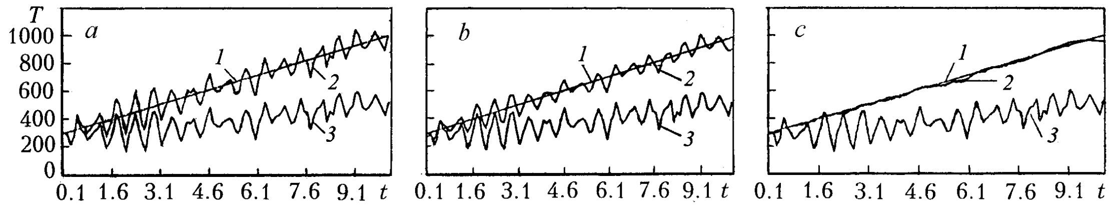

Fig. 2. Surface temperature recovered from a depth of 0.1 mm without regularization (a) and with regularization parameters $\alpha = 10^{-8}$ (b) and $\alpha = 10^{-7}$ (c): 1 and 2) initial and recovered surface temperatures; 3) noised temperature at a depth of 0.1 mm. T, K; t, sec.

quasi-inversion with regularization. The time step was taken equal to 0.01 sec, the step along the coordinate — 0.0001 m. The regularization parameter was chosen experimentally.

Results of the numerical experiment shown in Fig. 2c indicate that if the temperature measured at some distance from the surface has a random additive error, then direct recovery of the surface temperature is accompanied by the appearance of a similar error in the temperature under recovery. Use of the regularization algorithm, which is constructed based on the method of quasi-inversion, for recovery of temperature leads to instability of the computational process and accumulation of error on approaching the specimen surface. Oscillations in the solution can be suppressed by correct (rational) choice of the regularization parameter α. In our case, the value of the regularization parameter $\alpha = 10^{-7}$ is rational.

An analysis of the results obtained allows us to draw a conclusion on the stability of the algorithm suggested and on the possibility of its use in solution of problems of recovery of the surface temperature of the copper electrode by the data of temperature measurements at a certain depth.

Further, the algorithm developed was used to identify thermal processes on the surface of the steel part in its recovery by electrical contact sintering of metal powders. It is shown in [5] that the data of the calculation are in good agreement with actual values of temperature obtained on the surface of the part.

In conclusion, we note that improvement of the accuracy of identification of thermal processes on the surface of the electrode and the part, which is reached by the algorithm suggested, allows one to improve the accuracy of prediction of regimes in development of new technologies of electrical contact sintering.

NOTATION

x, coordinate, m; t, time, sec; T, temperature, K; q, temperature gradient, deg/sec; q_s, heat-flux density, W/m^2; t_f, time of occurrence of the studied process, sec; x_1, specified depth, mm; a, thermal diffusivity, m/sec^2; $T_0(t)$ and $q_0(t)$, temperature and its gradient measured at a specified depth, K and deg/sec; T_α, temperature in regularized solution, K; τ and h, steps of the computational grid along time and coordinate; α, regularization parameter; M, N, numbers of division of the computational grid. Subscripts: f, finite; s, cross-section area; 0, initial value.

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